SCATTERING REGIMES:  (Depends on ratio \( \lambda / L \))

High Frequency:  \( \lambda \ll L \), Optics like scattering, mostly independent scattering centers, angle of incidence = angle of reflection (backscatter, surface normal points back toward radar); Signature is coherent sum (phasor addition) of scattering centers.

Coherent Sum:  \[ \sigma = \sum_{i=1}^{N} \left[ \mathbf{\sigma}_i \exp(-j2k \cdot \mathbf{r}_i) \right]^2 \]

Incoherent Sum:  \[ \sigma = \sum_{i=1}^{N} \mathbf{\sigma}_i \]

Resonant Region:  \( \lambda \approx L \). Surface traveling, edge, and creeping waves become important scattering mechanisms. Must have component of E in direction of propagation. Grazing angle phenomena. Amplitude at \( \theta_{\text{max}} = 49\sqrt{\lambda / L} \) degrees.

Maximum amplitude depends on aft reflection coefficient and surface impedance, usually less than \( 3\lambda^2 \) for PECs.

Low Frequency (Rayleigh Region):  \( \lambda \gg L \). Induced dipole moment. Scattering proportional to (frequency)\(^4\).

\[ \sigma = (4 / \pi)k^4 \lambda^2 \]

CONSTANTS:

\[ \varepsilon_0 = 8.85 \times 10^{-12} \text{ Farads / meter} \]

Permittivity:  \[ \varepsilon = \varepsilon / \varepsilon_0 = \varepsilon' - j\varepsilon'' \]

\[ \varepsilon'' = \sigma / (\omega \varepsilon_0) \]

Permeability:  \[ \mu_0 = 4\pi \times 10^{-7} \text{ Henrys/meter} \]

\[ \mu_r = \mu / \mu_0 \]

Free space wave impedance:  \[ Z = 120\pi = 377 \text{ ohms} \]

Velocity of Propagation:  \[ c = (\varepsilon_0 \mu_0)^{-1/2} = 3 \times 10^8 \text{ m/s} \]

Index of Refraction:  \[ n = \sqrt{\varepsilon \mu_r} \]

Radian Frequency:  \[ \omega = 2\pi f \]

Wave Vector:  Points in direction of propagation, \( |k| = 2\pi / \lambda \)

Frequency and Wavelength relationships:

\[ f \cdot \lambda = c ; \quad \omega \cdot \mu_0 = 240\pi^2 / \lambda = 2367 / \lambda ; \quad \omega \varepsilon_0 = k / \eta = 1 / (60\lambda) \]

MATERIAL CHARACTERIZATION:

Bulk Impedance:  \[ Z = \eta \frac{\mu_r}{\varepsilon_r} \]

Reflection coefficient depends on polarization. For normal incidence it is  \[ R = (Z - \eta) / (Z + \eta) \]. dB = 20 log10(R)

Surface impedance of material backed by ground plane, normal incidence:

- General Case:  \[ Z = \eta \frac{\mu_r}{\varepsilon_r} \tan h(jk / \varepsilon_r \mu_r d) \]
- Thin layer:  \[ Z = j \eta \mu_r k d, \text{ independent of } \varepsilon_r \]

Resistive Layer, Ohms per Square:  \[ R = 1 / \eta \sigma \]

Impedance of thin layer, Ohms per Square:

\[ Z = -j \eta / [k(e_r - 1) \tau] \]

Bulk Loss Characterization:  \[ \varepsilon'' = \sigma / (\omega \varepsilon_0) = 60\lambda \sigma = 60\lambda / \rho \]

MEASUREMENTS:

Radar Equation:

\[ P_r = P_i G_i G_r R \frac{2 \sigma}{(4\pi)^3 R^4} \]

Noise Power:  \[ P_n (dBm) = -114dBm + 10 \log R_{MHz} + NF_{dB} \]

Ground Plane Range:

\[ h_{\text{antenna}}h_{\text{target}} = \frac{R}{\lambda} / 4; \quad \text{Ideal peak gain} = 16 = 12dB; \quad \text{Far Field distance} = 2D^2 / \lambda. \]

Down Range Image:

Resolution \( \Delta r = c / (2B) = (\lambda / 2) / (\Delta f / f) \), Extent = N \Delta r

Cross Range Image:  (isar)

Resolution \( \Delta r = \lambda / (2\Delta \theta) \)

DATA TYPES:

Probability Density Function (PDF):  Probability P(\sigma) that \sigma lies between \sigma and \sigma + d\sigma (histogram).

Cumulative Distribution Function (CDF):  \[ CDF(\mathbf{\sigma}) = \int_{-\infty}^{\mathbf{\sigma}} P(\mathbf{\sigma}) \text{d}\mathbf{\sigma} \]

Median:  \[ CDF(\mathbf{\sigma}_m) = 0.5 = \int_{-\infty}^{\mathbf{\sigma}_m} P(\mathbf{\sigma}) \text{d}\mathbf{\sigma} \]

ANALYTICS:

Specular Reflection:  \( \theta_r = \theta_i \)

Transmission (Snell’s Law):

\[ k_{\text{inc}} \sin \theta_{\text{inc}} = k_{\text{tran}} \sin \theta_{\text{tran}} \]

Geometric Optics (specular scattering):

\[ \sigma_{\text{go}} = \pi R_1 R_2 / R_1 + R_2 \] radii of curvature at specular point

\[ E^s = R \left( \frac{\mathbf{p}_1 \cdot \mathbf{p}_2}{2 (\mathbf{p}_1 \cdot \mathbf{s}) (\mathbf{p}_2 \cdot \mathbf{s})} \right) D \cdot E^i \exp(-jk s) \]

Physical Optics (specular and end region scattering):

Current:  \[ J_{\text{po}} = \int_{-h}^{h} \mathbf{H}^{\text{inc}} \times \mathbf{\nabla} g \text{ dS} \]

Field:  \[ \mathbf{H}^{\text{scat}} = \int (\mathbf{J}_{\text{po}} \times \nabla g) \text{ dS} \]

Electric and Magnetic Field Integral Equations:

\[ \mathbf{E}^{\text{tot}} = \mathbf{E}^{\text{inc}} + \int \left\{ -j \mathbf{k} \cdot \mathbf{J} + \frac{\mathbf{\rho}}{\varepsilon} \times \mathbf{V} \right\} \text{dS} \]

\[ \mathbf{H}^{\text{tot}} = \mathbf{H}^{\text{inc}} + \int \left\{ -j \frac{\mathbf{k}}{\eta} \cdot \mathbf{J} + \frac{\mathbf{\mu}}{\mu} \times \mathbf{V} \right\} \text{dS} \]

\[ \theta = 1 \text{ for } r \in R_1; \quad 1/2 \text{ for } r \in \partial R_1; \quad 0 \text{ otherwise} \]

Current Continuity:  \[ \nabla \cdot \mathbf{J} = \frac{\partial \sigma}{\partial t} \]

Units:  Square Meters dBm

1000  30  \sigma_{\text{dBsm}} = 10 \log_{10} \sigma_{\text{sm}}

100  20

10  10  \sigma_{\text{sm}} = 10 * (\sigma_{\text{dBsm}} / 10)

1  0

0.1  -10

0.01  -20
Green’s Functions:

\[
\begin{aligned}
\mathbf{g} &= \exp(-j \mathbf{k} \cdot \mathbf{R}) / 4\pi R \\
\mathbf{V}_g &= (1 - j \mathbf{kR}) \mathbf{R} g / R^2
\end{aligned}
\]

3D:

\[
\begin{aligned}
\mathbf{g} &= \frac{1}{4j} H(2)(\mathbf{k}) \\
\mathbf{V}_g &= \frac{k}{4j} H(2)(\mathbf{k}) \mathbf{p}
\end{aligned}
\]

Method of Moments:

\[
Z_{ij} = < W_i, L(J_j) > = + j \eta \iint (W_i \cdot J_j - (\mathbf{V} \cdot W_i)(\mathbf{V} \cdot J_j)) / k^2 g dS_i dS_j
\]

Unit Vectors:

\[
\mathbf{u} = [\cos(\theta) \cos(\phi), \cos(\theta) \sin(\phi), -\sin(\theta)]
\]

\[
\mathbf{k} = [-\sin(\phi) \cos(\theta), \cos(\phi), 0]
\]

HIP POCKET ESTIMATION FORMULAS: (for backscatter, \( \lambda < L \))

Scattering Mechanisms:

- Specular, end region, leading and trailing edge diffraction, surface traveling, creeping, and edge waves, multiple bounce, tip diffraction, etc.

Constant Phase Region (specular return):

\[
\sigma_{specular} = 4\pi (Effective\ Area)^2 / \lambda^2
\]

Characteristic Dimension of Constant Phase on Curved Surface:

\[
L_{effective} = \sqrt{(R\lambda / 2)}
\]

where \( R \) = radii of curvature at specular point

Specular (Peak) Returns (polarization independent):

Planar Surfaces:

\[
\sigma = 4\pi A^2 / \lambda^2
\]

Singly Curved Surface:

\[
\sigma = k R L^2
\]

Doubly Curved Surface:

\[
\sigma = \pi R_1 R_2 R_3 \text{ at specular point}
\]

Approximate Beam Width:

\[
\Delta\theta = 57\lambda / L \text{ (rect. distribution)}
\]

Flat Plate Formulas (sides \( a, b, \) or \( L \)):

\[
\sigma = [4\pi(ab)]^2 / \lambda^2 \cos^2(\theta)[\sin(P) / P]^2 [\sin(Q) / Q]^2
\]

where \( P = ka \cos(\theta) \sin(\theta), Q = kb \sin(\phi) \sin(\theta) \)

Envelope \( \downarrow \) to Edge of Length \( L: \) \( \sigma = L^2 \cot^2 \theta / \pi \)

Envelope Along Diagonal \( (4\pi^2): \) \( \sigma = \lambda^2 \cos^2 \theta / (\pi^3 \sin^4 \theta) \)

Edge Diffraction:

Leading Edge \( \parallel \) parallel, Trailing Edge \( \perp \) perpendicular:

\[
\sigma = L^2 / \pi
\]

Curved Edge:

\[
\sigma = R\lambda / 2\pi
\]

Corner Reflectors:

Dihedral:

\[
\sigma = 8\pi(ab)^2 / \lambda^2
\]

Trihedral:

\[
\sigma = 12\pi b^4 / \lambda^2
\]

Surface Traveling and Edge Wave (component of \( E \) in direction of propagation on surface):

Location:

\[
\theta = 49(\lambda / L)^{1/2}
\]

Amplitude:

Depends on aft reflection coefficient and surface impedance along propagation path, usually \( \leq 3\lambda^2 \)

RCS of HOLES and SLOTS:

Use Babinet’s principle: Interchange \( E \) & \( H \), use complementary geometry (e.g., slot to wire, hole to disk), then use existing analytical approaches and computer codes for Rayleigh, resonant, or optical regimes.

Small holes of radius \( a, ka < 1: \)

\[
E_{\parallel} \text{ plane of hole}: \quad \sigma = (16 / 9\pi)^2 (ka)^2 [2 + \sin^2 \theta]^2
\]

\[
E_{\perp} \text{ plane of hole}: \quad \sigma = (4^3 / 9\pi)^2 (ka)^2 \cos^4 \theta
\]

Slots: Traveling wave for \( H \) in direction of slot

Complementary Impedance:

\[
Z_{comp} = 377^2 / (4Z)
\]

Radar Cross Section Reference

Marietta Scientific, Inc.

John Shaeffer, Brett Cooper

376 Powder Springs St., Suite 240A
Marietta, Georgia 30064
(770) 425-9760
www.MariettaScientific.Com

Radar Cross Section \( \sigma \): A measure of power reflected by a target. Units are square meters (area). When expressed in dB, the reference is 1 square meter, \( dBm \) or \( dBm^2 \). Make non-dimensional by normalizing to wavelength, \( \sigma / \lambda^2 \).

Monostatic Cross Section: RCS in the backscatter direction, i.e., receiver at same location as transmitter. Usual case.

Bistatic Cross Section: RCS in direction other than backscatter, i.e., receiver at different location than transmitter.

Polarization:

Direction of \( E \) vector for transmit or receive. Typically vertical or horizontal.

Wave Vector \( k \): Vector direction of propagation for EM wave. Scalar magnitude inversely related to wavelength, \( k = 2\pi / \lambda \).

EM Wave: Propagation of electromagnetic energy. Has electric and magnetic vector components, \( E \) and \( H \), and direction of propagation \( k \). \( E \), \( H \), and \( k \) are mutually orthogonal. EM wave characterized by: wavelength or frequency, direction of propagation, and polarization of \( E \). Wave impedance \( \eta = E/H = 120\pi = 377 \) ohms in free space.

Specular Point, Flash Point, Region of Stationary Phase: Point on scattering body where angle of incidence is equal to angle of reflection. For back scatter, this is where the surface normal points back toward the radar.

Scattering Center: Region of body which reflects EM energy (hot spot), e.g. specular points, multiple reflection, surface wave, or diffraction locations.

Surface Wave: Non specular scattering mechanism dominant for resonant region bodies, \( L/\lambda < 10 \). Types are traveling, creeping, or edge wave which occur near grazing incident angles when the incident \( E \) field has a vector component along the body in direction of propagation.

<table>
<thead>
<tr>
<th>Band</th>
<th>Frequency</th>
<th>Wavelength</th>
</tr>
</thead>
<tbody>
<tr>
<td>HF</td>
<td>5-30 MHz</td>
<td>200-33 ft.</td>
</tr>
<tr>
<td>VHF</td>
<td>50-300 MHz</td>
<td>18-3 ft.</td>
</tr>
<tr>
<td>UHF</td>
<td>300-1000 MHz</td>
<td>3-1 ft.</td>
</tr>
<tr>
<td>L</td>
<td>1-2 GHz</td>
<td>1-0.5 ft.</td>
</tr>
<tr>
<td>S</td>
<td>2-4 GHz</td>
<td>6-3 in.</td>
</tr>
<tr>
<td>C</td>
<td>4-8 GHz</td>
<td>3-1.5 in.</td>
</tr>
<tr>
<td>X</td>
<td>8-12.5 GHz</td>
<td>1.5-0.9 in.</td>
</tr>
<tr>
<td>K 1</td>
<td>12.5-18 GHz</td>
<td>0.9-0.66 in.</td>
</tr>
<tr>
<td>K 2</td>
<td>26.5-40 GHz</td>
<td>0.45-0.3 in.</td>
</tr>
</tbody>
</table>